Marine Environmental
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A Discussion of Procedures to Estimate Extreme Wave Heights over the Canadian Atlantic Continental Shelf

J.S. Readshaw and W.F. Baird
W.F. Baird & Associates, Coastal Engineers, Ltd.

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DEPARTMENT OF FISHERIES AND OCEANS
OTTAWA, CANADA

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ABSTRACT

A general review of the state of the art of estimating extreme values of the significant wave height is provided. The general types of analyses that involve recorded wave data, hindcast wave data, and project storms are discussed.

The use of asympotic probability distributions are reviewed in detail. In particular, the assumptions and limitations of these procedures are described.

The characteristics of the available data and the limitations that these characteristics impose on extreme value statistics are discussed at length. The climatology of the Atlantic Continental Shelf is briefly described, and unique requirements of extreme value procedures for this area are noted.

Finally, recommendations for further studies necessary to develop reliable procedures for estimating extreme wave statistics over the Canadian Atlantic Continental Shelf are made.

RESUME

On passe en revue les plus récentes méthodes d'estimation de la valeur extrême des hauteurs de vagues importantes. Les types généraux d'analyse qui portent sur des données enregistrées,des rétroprévisions et des tempêtes de projet sont examinés.

L'utilisation des distributions de probabilité asymptotiques est examinée en détail. On décrit en particulier les hypothèses et les limites de ces méthodes.

Les caractéristiques des données disponibles et les limites que ces caractéristiques imposent aux statistiques sur la valeur extrême sont développées longuement. La climatologie du plateau continental atlantique est brièvement décrite et les conditions uniques de la détermination des valeurs extrêmes pour cette région sont exposées.

Pour terminer, on formule des recommandations sur les futures études qui seraient nécessaires pour élaborer des méthodes fiables d'établissement de statistiques sur la hauteur extrême des vagues sur le plateau continental de l'Atlantique canadien.

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1 SYNOPSIS

This report was prepared as a first phase in the development of procedures to estimate extreme wave statistics required for the recovery of oil and gas over the Canadian Atlantic Continental Shelf.

The report provides a general review of the state of the art of estimating extreme values of the significant wave height. The general types of analyses that involve recorded wave data, hindcast wave data, and project storms are discussed.

The use of asymptotic probability distributions are reviewed in detail. In particular, the assumptions and limitations of these procedures are described.

The characteristics of the available data and the limitations that these characteristics impose on extreme value statistics are discussed at length. The climatology of the Atlantic Continental Shelf is briefly described and unique requirements of extreme value procedures for this area are noted.

Finally, recommendations for further studies necessary to develop reliable procedures for estimating extreme wave statistics over the Canadian Atlantic Continental Shelf are made.

2 INTRODUCTION

Estimates of extreme wave height, the associated wave period and the elevation of the wave crests above mean water level are required for the design of offshore structures to be used in the exploration and production of oil and gas on the Canadian Atlantic Continental Shelf. Estimates of these extreme values are necessary to ensure the safety of the structures. Extreme wave statistics have been used in the past for the design of exploratory and production structures to determine:

a) the extreme loading including drag, inertial and impact forces on offshore structures

b) the minimum deck or freeboard clearances for moored or gravity structures.

c) the response of floating structures in survival conditions

Reliable estimates of extreme wave statistics are necessary primarily for the design of safety and survival aspects of offshore structures. The operational and dynamic design of the structures is based to a much greater extent on waves of lower height but with a greater probability of occurence. The determination of the probabilities of parameters which describe the more persistant wave conditions require a separate analysis from the extreme wave statistics analysis.

The wave parameters which are required from an extreme value analysis include:

(a) the expected extreme significant wave height and its period for a given level of risk (usually expressed as a return period).

(b) The expected maximum wave height and period which occur during the storm. This wave height depends on the duration of the storm peak, the period of the wave and the distribution of waves within the storm.

(c) the height of the maximum wave crest above the mean sea surface.

(d) the direction of the extreme waves. The direction of the extreme waves is required for very large gravity type structures which may be required on the Canadian Atlantic Continental Shelf to withstand ice forces. This parameter has not generally been included in extreme value studies in the past. Direction is only available where a hindcast procedure is used.

This report discusses the state of the art of procedures to estimate the extreme significant wave height. Procedures for

determining the expected maximum wave, the wave periods and the height of the wave crest are separate problems and a discussion of current procedures is beyond the scope of this report. The role which those parameters play in the estimation of extreme wave parameters is introduced in context in this report but further study is required before a procedure can be defined which is valid and reliable for the Canadian Atlantic Continental Shelf area.

3 RISK AND THE ESTIMATION OF EXTREME WAVE STATISTICS

In defining the extreme wave values there are two concepts of risk which are of interest to the engineer. These concepts are referred to as the return period and the non-encounter probability. The two concepts are related but the distinction between them is important.

The return period is defined as the average waiting time between occurrances of a wave with a height equal to or greater than a specific wave height H. For example, it is common practice to estimate the significant wave height associated with a return period of 100 years. On average, this wave height is expected to be exceeded only once every 100 years. However, there is a risk that this wave height will be exceeded more often than once every 100 years. This risk must be conveyed to the design engineer and evaluated in quantative terms. The risk can be defined by the non-encounter probability.

The non-encounter probability is defined as the probability during a specific interval or design life that the largest wave height occurring during the interval or life is less than or equal to a specific wave height H. The acceptable degree of risk associated with the non-encounter probability is a subjective decision which must be made by the design engineer. A more rigorous definition and derivation of the return period and non-encounter probability is given by Borgman and Resio (1981) and Borgman (1963).

The return period R and the non-encounter probability NE(X) are related in the following manner. In a series of stationary, independent and identically distributed observations (X1,X2, ...Xn), the probability that Xi is less than or equal to an arbitrary value x is defined as Fx(x), where the value of Fx(x) is given by an asymptotic probability distribution of the value Xi .... Xn.

a) The return period R is then defined as:

$$R = \frac{1}{1 - Fx\ (x)} \qquad (3.1)$$

b) The non-encounter probability is defined as:

$$NE(x) = \left[1 - (1 / R)\right]^{n} \qquad (3.2)$$

where n = the design life of the structure, defined as:

i) the number of years, if the structure is designed for an annual maximum event

ii) the number of events, if the structure is designed for events which occur more often than once a year.

The distinction between cases (i) and (ii) is important. Case (i) is valid only when the structure is designed to withstand an event which occurs only once a year. An example is the annual spring freshet in hydrology. Case (ii) is valid when more than 1 event occurs annually. For example, if 4 storms occur each year, then over a 10 year life, n = 40.

Using equations 3.1 and 3.2 one can determine the wave height associated with a specific return period, given a defined acceptable risk of the event occurring. This wave height can be calculated, knowing the asymptotic probability distribution of the extreme waves, and the acceptable risk, by the following expression:

$$[Fx(x)]^n = NE(x) \qquad (3.3)$$

Equations 3.1, 3.2, and 3.3 are valid only for cases i and ii. The return period and the non-encounter probability for the physically more realistic case where a random number of storms occur during the life of a structure is given in Appendix 1.

The previous discussion is intended to introduce the concept of risk to the extreme value problem. An acceptable level of risk can only be defined by the designer or the regulatory body. Once the risk is defined then an estimate of the extreme wave value can be made. The remainder of this report discusses the state of the art of estimating the extreme sea state. In terms of equations 3.1, 3.2, and 3.3, the following discussion centers only about the problems of defining the asymptotic probability distribution of large sea states specified by Fx(x) in equations 3.1 and 3.3. The uncertainty associated with this estimate is also introduced. It is important to note that this uncertainty is separate from the concept of risk introduced above. The uncertainty results from the choice of the asymptotic distribution, the extent and quality of data and the method of fitting the distribution to the data.

4 PROCEDURES TO DETERMINE EXTREME WAVE STATISTICS

There are three procedures which can be used to estimate extreme wave statistics. These procedures are:

a) The extrapolation of recorded wave data using a asymptotic probability distribution function to extrapolate to return periods greater than the length of record of the original data.

b) The modelling of the wave generation process to develop a wave climate for the area of interest, using recorded meterological data (isobaric pressures or wind speed) followed by the extrapolation of the developed wave data, using an asymptotic probability distribution.

c) The development of a statistical model of storms for the area from which the parameters of the return period storm are derived. The waves in this storm are hindcast using a suitable numerical procedure.

4.1 Extrapolation of Recorded Wave Data

Estimates of extreme wave statistics can be obtained by fitting recorded wave data to an asymptotic probability distribution and extrapolating the distribution to determine the extreme waves for a given return period. This method has been used in many applications including the North Sea, Carter and Challenor (1978), where 7 years of recorded data were available. The asymptotic probability distributions and both their practical and theoretical limitations are discussed in Chapters 5 and 6. The major limitation of this procedure which is generally the length of coverage of recorded data which is available is discussed in Chapter 6.

4.2 Hindcasting and Extrapolation of Storms

The limitations of using recorded wave data can be avoided by hindcasting waves during storms in the area of interest. This procedure takes advantage of the larger periods of coverage which are available for meteorological data. The estimates of extreme wave values are determined by extrapolating a wave height which is characteristic of the peak of a storm. Previous studies have used either the maximum wave height or the significant wave height during the peak of the storm. Extrapolation is made by assuming a suitable asymptotic probability distribution. This approach has been used by both private industry and regulatory agencies in the North Sea (Haring 1979; Ewing, Weare, and Worthington 1979) in the Gulf of Mexico, off the east coast of the United States (Ward, Evans and Pompa, 1978), in the Great Lakes (Resio 1973) and off the coast of California, Petraukas and

Aagaard (1971).

There are three methods which have been used to translate hindcast storm waves into estimates of extreme wave values.

1) The hindcast storm waves are assumed to be only one of an infinite number of samples of the total population of storms for a given location. The parameters of the probability distribution are estimated from the sample and are used to extrapolate to longer return periods. This procedure requires the assumption of two probability distributions. The probability distribution of storms is assumed to be defined by a discrete probability distribution such as the Poisson distribution. The distribution of waves within the storm is assumed to be one of many asymptotic probability distributions discussed in Chapter 5.0. The intensity of the storm is characterized by the significant wave height. The maximum wave height during a storm with a given return period is calculated from the significant wave height for the same return period using the methods of Cartwright and Longuet-Higgins (1956) or a modification of their distribution suggested by Haring, Osborne and Spencer (1976), Forristall (1978), Longuet-Higgins (1980) and Walden and Prescott (1980). These methods have not been reviewed in this study and further review should be undertaken prior to using one of the procedures.

2) The hindcast storm waves are assumed to represent the true population of storm waves. In this procedure, storm waves are not extrapolated beyond the largest storm in the hindcast. This method is based on the premise that there is a finite limit to the intensity of a storm which may occur and therefore it is erroneous to extrapolate to long return periods. This method has been used by Haring (1979) as a second estimate of extreme sea states in the North Sea. The method is very sensitive to the tracks of storms over the hindcast period and may not be applicable to tropical storm systems where the synoptic scale of the storm is small and the track highly variable. The maximum wave height during the most severe storm is calculated using the same procedures referenced for Method 1.

3) In the third method, the expected maximum wave in each storm is calculated from the hindcast sea state and extrapolated as in Method 1 in place of the significant wave height. This method has the disadvantage that the extreme values are only representative of conditions when the storm peak is of the same duration assumed in the calculation of the maximum wave within each storm. There is no reason why this should be true. The maximum wave height in each storm is calculated by the same procedures discussed for procedure 1.

4.3 Statistical Models of Storms

There are two possible methods for this procedure. If sufficient recorded meteorological data is available, statistics of several

storm types, defined by either their origin or their storm track or both, can be developed and the waves produced by the worst storm are hindcast using a verified numerical procedure. In this approach the dependency of storms on both synoptic scale and large scale meteorological features in the atmosphere can be modelled. The procedure requires the use of Bayesian statistics which have not previously been used to develop extreme wave statistics. This type of approach has only recently been suggested; Resio (1978), but may have application for the Canadian Atlantic Continental shelf.

If recorded meteorological data is not available to develop the statistics of the storm type, it may become necessary to estimate the probable maximum storm for the area. This approach is recommended by the World Meteorlogical Organization, WMO (1973) when recorded data is either scarce or of short duration. This approach is currently used by hydrologists to predict extreme flood events.

4.4 Summary

The first two procedures outlined above require long record lengths of either recorded wave data or meteorological data. The length of recorded wave data for the first procedure or of meteorological data for the second procedure should be approximately one half the return period for which an estimate of the extreme wave is desired. In the second procedure recorded wave data is also required to verify the hindcast procedure. Sufficient recorded data must be available to verify that the hindcast represents the wave generation process for all types of storms which may occur. Over the Canadian Atlantic Continental Shelf both extratropical or tropical cyclones may produce storm waves and two hindcast procedures, one for each type of cyclone, may be required.

Procedures 1 and 2 have found widespread application for estimating extreme wave statistics. Both procedures require the use of an asymptotic probability distribution to extrapolate to long return periods and because the extrapolation is sensitive to the choice of distribution, the quality of data used and the method of fitting the distribution, the emphasis in this report has been placed on these three principal items. In Chapter 5.0 asymptotic distributions, the method of fitting the distributions to the data and the estimation of uncertainty due to the fit are discussed. Chapter 6.0 discusses practical limitations of the first two procedures. In Chapter 7.0 the climatology of the Canadian Atlantic Continental Shelf is discussed and in Chapter 8.0 the project storm approach is discussed. Recommendations for a suitable procedure to develop extreme wave statistics are made in Chapter 9.0.

5 STATISTICS OF EXTREME VALUES

5.1 Asymptotic Probability Distributions

The estimation of extreme wave statistics by either one of the first two procedures outlined in Chapter 4.0 requires the extrapolation of data whenever the required return periods are longer than the length of coverage of the data.

Previous studies to estimate extreme wave statistics have used a type of asymptotic probability distribution known as Fisher Tippett type distributions. Fisher and Tippett (1928) showed that the distributions of maxima or minima of stationary, independent and identically distributed variables must approach one of three asymptotic distributions known as the Type I, Type II, or Type III distributions. An asymptotic probability distribution defines the probability of the largest or the smallest value in a random number of samples. The samples are drawn from a population which can also be described by some other non-asymptotic initial probability distribution. Asymptotic probability distributions which have been used in the literature for the estimation of extreme wave statistics are listed in Table 1. The distributions are categorized by limiting type and a partial list of references is provided. The references are not exhaustive but consist only of those reports where the application of the particular distribution has been discussed in detail. A more complete list of references is provided in the bibliography included in this report in Appendix 4.

There are other asymptotic distributions which have been used by hydrologists to estimate extreme flood events but which have not been applied to extreme wave studies. These distributions include the Log-Pearson type distributions and the three parameter log-normal distributions. Further study would have to be undertaken to determine if these distributions are also valid for estimating extreme wave statistics.

The three Fisher Tippett limiting distributions are characterized by the type of initial distribution from which the maxima or minima are drawn, and by the limits on the asymptotic distribution. The general form of the three limiting types for the largest values (maxima) from an initial distribution are discussed below. The initial distribution, the parameters of the asymptotic distribution and the limiting bounds on the asymptotic distribution are summarized. In Figure 1, the probability density function, and the cumulative probability distribution for Type I, Type II, and Type III asymptotic distributions are illustrated. The distributions have been plotted using wave data hindcast by the U. S. Army Corps of Engineers, Waterways Experiment Station for a location approximately 240 km northeast of St. John's, Newfoundland. These hindcast data are discussed more fully by Baird and Readshaw (1981). The distributions which are shown in Figure 1 are asymptotic distributions of the largest value. Asymptotic distributions of the smallest value in a

Distribution	Type I	Type II	Type III	References (Partial List)
Double Exponential (Gumbel)	$P[H \leq X] = \exp[-\exp\{\frac{X-B}{A}\}]$			Petrauskas and Aagaard (1971) Copeiro (1978) Haring (1979) Ewing et al (1979) Carter and Challenor (1981)
Three Parameter Double Exponential	$P[H \leq X] = \exp[-(X-C)\cdot\exp\{\frac{X-B}{A}\}]$ or $P[H \leq X] = \exp[-\exp\{[\frac{X-B}{A}]\cdot(X-C)\}]$			Copeiro (1978)
Two Parameter Log Normal	Cumulative Probability function cannot be expressed in closed form			Copiero (1978)
Three Parameter Log Normal	Cumulative Probability function cannot be expressed in closed form			Copiero (1978)
Type II distribution		$P[H \leq X] = \exp[\{\frac{X-B}{A}\}^{-C}]$		Thom (1973) Gumbel (1958) Resio (1978)
Weibull Distribution			$P[H \leq X] = 1-\exp[-\{\frac{X-B}{A}\}^{C}]$	Petrauskas and Aagaard (1971) Copeiro (1978)
Type III Distribution			$P[H \leq X] = \exp[-\{\frac{B-X}{A}\}^{C}]$	Gumbel (1958) Suarez Bores (1974)
Exponential			$P[H \leq X] = 1-\exp[-\{\frac{X-B}{A}\}]$	Copeiro (1978)
Gamma Distribution	The original reference is in Japanese and details are not available			Yamaguchi et al (1978)
Log Pearson Type III Distribution	Details not available as to which type the Log Pearson III distribution should be included under.			Resio (1977)

NOTE: The parameters in each distribution are given in the form used for estimating by linear regression

TABLE 1

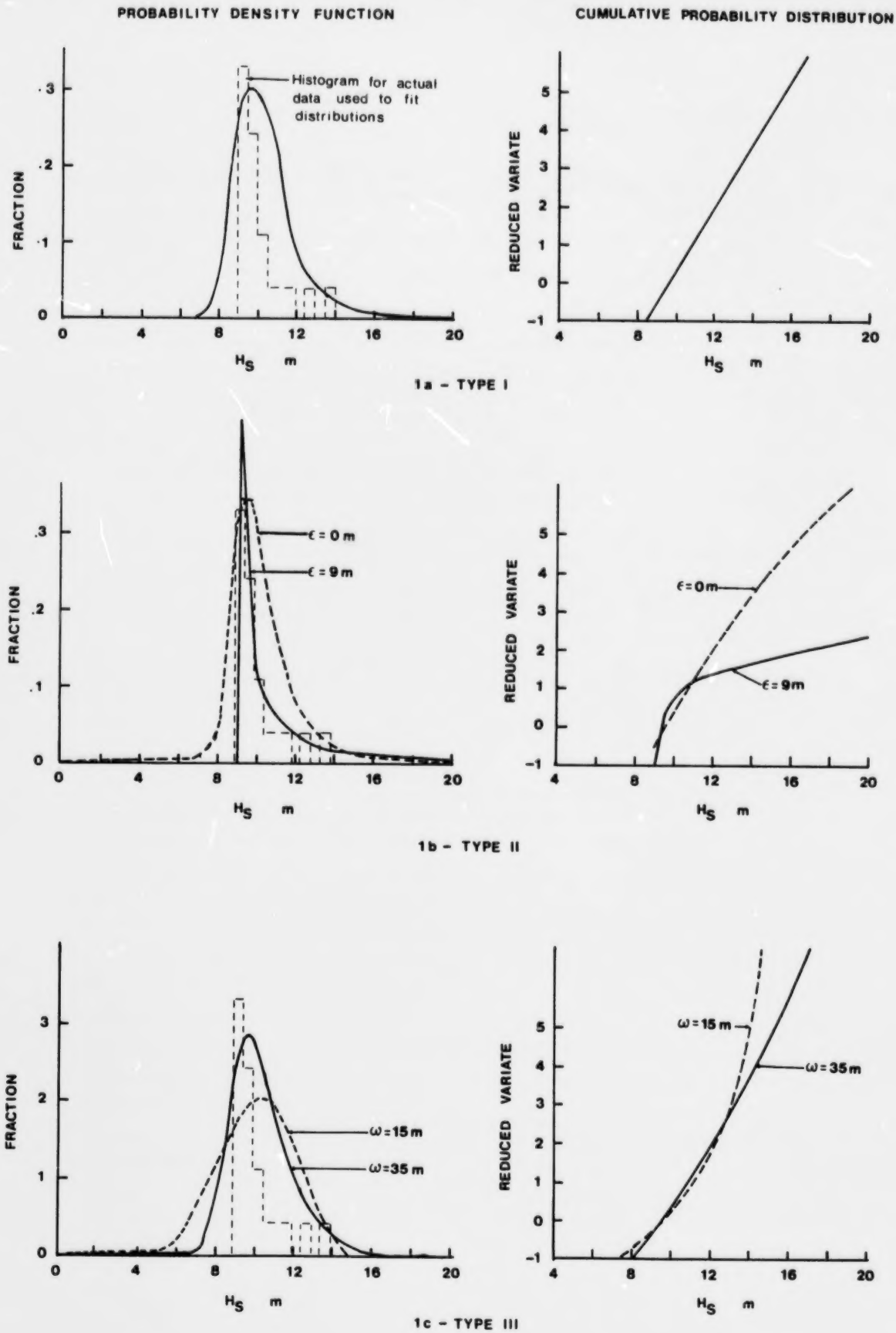

PROBABILITY DENSITY FUNCTION
CUMULATIVE PROBABILITY DISTRIBUTION
Histogram for actual data used to fit distributions
FRACTION
REDUCED VARIATE
H_S m
1a - TYPE I
$\epsilon = 0$ m
$\epsilon = 9$ m
$\epsilon = 0$m
$\epsilon = 9$m
1b - TYPE II
$\omega = 15$ m
$\omega = 35$ m
$\omega = 15$m
$\omega = 35$m
1c - TYPE III

FIGURE 1

random number of samples are related to these distributions by a simple principle of symmetry. Reference should be made to Gumbel (1958) for more details.

5.1.1 Type I - Asymptotic Distribution

$$Fx(x) = \exp[-\exp[-\alpha(x - u)]] \qquad (5.1)$$
$$-\infty < x > \infty$$

The maxima in a Type I asymptotic distribution are drawn from a population having an initial exponential distribution (Gumbel, (1958), p. 275). The distribution is unlimited in both directions, Figure 1a, and has two parameters, α and u, which must be estimated. The parameter α is a measure of the dispersion of the distribution. The parameter u is the mode of the distribution.

Type I distributions which have been used to estimate extreme wave statistics include the double exponential distribution and a variation suggested by Copeiro (1978). The double exponential distribution is one of the most widely used asymptotic distributions for estimating extreme wave statistics.

5.1.2 Type II - Asymptotic Distribution

$$Fx(x) = \exp\left[-\left\{\frac{v}{x - \epsilon}\right\}^k\right], \; x \geq \epsilon \qquad (5.2)$$
$$= 0, \; x < \epsilon$$

The maxima in a Type II asymptotic distribution are drawn from an initial distribution known as Pareto's distribution or from a Cauchy distribution (Gumbel (1958), p 276).

The Type II distribution is limited on the left hand side, Figure 1b, and has three parameters, k, v, and ϵ, which must be estimated. In general, one of the three parameters must be chosen arbitrarily while the remaining two are estimated from the sample. The parameter k is an order of the moments of the initial distribution, v is the expected largest value of the initial distribution, and ϵ is the bound on the left hand side.

Thom (1973) has used this distribution to estimate extreme wind and wave statistics based on the fact that both wind speeds and wave heights cannot have a value of less than zero.

5.1.3 Type III - Asymptotic Distribution

$$Fx(x) = \exp\left[-\left\{\frac{w - x}{w - v}\right\}^k\right], \; x \leq w, \; w > 0 \qquad (5.3)$$
$$= 1, \; x > w$$

The maxima in a Type III asymptotic distribution are drawn from an initial distribution known only as a limited initial distribution (Gumbel 1958, p 277). The definition of a limited initial distribution is not provided in the literature.

The Type III distribution is limited on the right hand side, Figure 1c, and has three parameters, k, v, and w, which must be estimated. In general, one of the three parameters must be chosen arbitrarily while the remaining two are estimated from the sample. The parameter k is defined as the order of the lowest derivative of the probability function which does not vanish at x = w, v is the expected largest value of the initial distribution and w is the right hand bound of the initial distribution.

For the Type III distribution, the right hand bound w may be representative of the case where either the fetch or the depth limit the height of waves to some finite value.

The most common application of a Type III distribution is the Weibull distribution. This distribution is actually the asymptotic distribution of the smallest value in an initial population in which case the distribution is bounded on the left hand side. The Weibull distribution has been used in both its two parameter and three parameter forms. As a two parameter distribution with w = 0 and k = 2, the distribution is known as the Rayleigh distribution and is most often used to describe the continuous distribution of waves within a storm. However, Petrauskas and Aagaard (1971) used the Rayleigh distribution to estimate extreme waves for a study in which seven values of the parameter k, including k = 2, were used for the Weibull distribution. The three parameter Weibull distribution has found more widespread use and together with the double exponential distribution (Type I) are the most commonly used distributions for estimating extreme wave statistics.

There appears to be a logical flaw in the use of the Weibull distribution for estimating extreme waves. This flaw is discussed in more detail in Appendix 2. Further study of the Weibull distribution and the general form of the Type III distribution should be undertaken before this distribution is used in a study of extreme wave statistics.

5.1.4 Application of Asymptotic Probability Distributions

There is no theoretical or practical basis on which the selection of the best distribution amongst those listed on Table 1 can be based. Copeiro (1978) and Resio (1977) have demonstrated that all three types of distribution may be fitted to a sample within the constraints of any goodness of fit test. Both studies found that the behaviour of the right hand tail of the distributions varies so widely that a conclusive decision cannot be made as to which distribution is most appropriate or acceptable. The choice of distribution is a completely subjective one, usually made on

the basis of which distribution fits the available data when it is plotted on probability paper.

A review of the literature shows that regulatory agencies for the North Sea offshore oil industry do not use any one distribution in particular nor is there any agreement as to which type of distribution should be used.

Det Norske Veritas, the agency charged with certifying offshore structures in the Norwegian sector of the North Sea, currently bases its estimates of extreme wave statistics on a fit of the Weibull distribution to observed significant wave heights. The Norwegian Meteorological Institute use the double exponential (Gumbel) distribution. The Norwegian Institute of Technology advocates using all distributions and accepting the average value. The Institute of Oceanographic Sciences, Britain, uses both the Double Exponential and the Weibull distribution to estimate extreme wave statistics.

The variation which may result in an estimate of extreme waves depending on which distribution is used can be illustrated with an example from the literature. Copeiro (1978) gives an example using wave data hindcast for Cabo Machichaco in the Bay of Biscay and for Valencia, both locations along the Spanish coast. The data were fit equally well (based on a goodness of fit test) by Type I, II, and III asymptotic distributions including the double exponential, Weibull, and log normal distributions. General forms of the Type II and Type III distributions were also used. For a 100 year return period, the estimated extreme significant wave heights varied by 7 m at Cabo Machichaco and 3 m at Valencia. The average values were approximately 13 m and 9 m respectively. Most of this variation is due to the Type II distribution. If the value predicted by that distribution is ignored the variation becomes less than 2 m in both cases.

In a similar example, for the California coast Petrauskas and Aagaard (1971) found a difference of 1 m between Type I and Type III (Gumbel and Weibull) distributions. In their study, the maximum expected wave height (not the maximum significant value) hindcast during 22 of the worst storms in 60 years were used for fitting the distributions.

In summary, the fundamental problem in estimating extreme values is that the initial distribution from which the largest values are drawn is unknown. It is, therefore, not possible to determine if the asymptotic distribution of extremes is a Type I, II, or III distribution. Furthermore there is no objective way of choosing the correct distribution or in fact knowing if a Fisher Tippett limit exists. The most reliable procedure available appears to be to fit all distributions listed in Table 1 to the sample of recorded or hindcast waves, to plot the fitted distributions and the data, and to determine confidence limits for the estimates of the distribution parameters which have been estimated from the sample. A comparison of the fitted

distribution and the plotted data, especially in the region of the higher values of wave height, will reveal any distributions which clearly should be rejected. A distribution can be rejected if the observed data departs noticeably from the fitted distribution. This comparison can only be made up to the highest observed value. No basis is available for accepting or rejecting a distribution beyond this point.

In the remainder of this chapter the initial assumption of stationary, independent, and identical distributed variables, the choice of a plotting position, and the estimate of the distribution parameters are discussed. Practical limitations of the first two procedures which were outlined in Chapter 4 are discussed in Chapter 6.0.

5.2 Assumption of Stationary, Independent and Identically Distributed Variables

It is generally stated that the Fisher Tippett asymptotic distributions are limited to observations which are stationary, identically distributed, and independent. In addition, the number of samples should be large and the initial distributions must be of one of the three limiting types.

The condition of stationarity is very seldom satisfied during storms. In practice, this requirement is generally ignored for extratropical storms. Waves during storms of tropical origin such as hurricanes are not stationary and other procedures are used for these storms. Secular variations in climate will also result in non-stationary conditions.

Borgman and Resio (1981) describe a procedure for observations which are both non-stationary and non-identically distributed. Battjes (1970), Durning (1971), Nolte (1973) and Yang, Tayfin, and Hsiao (1974) also describe other procedures for the case of non-stationary, non-identically distributed variables. These procedures have not been reviewed during this study. Further review should be undertaken prior to using one of these procedures.

The exact definition of identically distributed and independent observations is not clear in the literature. Gumbel (1958) states that observations of the largest values must be made under identical conditions and that the initial distribution and its parameters must be the same for each sample. Galambos (1978) states only that the assumption of independent and identical distributions may be a critical assumption and that if it is not a correct one, the asymptotic properties of the extremes may deviate significantly from the assumptions of an independent and identically distributed case. However, there are cases where the assumption of independent and identically distributed variables may give a good approximation. As an example, Galambos discusses the asymptotic properties of the strength of a sheet of metal or

of a bundle of threads where the strength of the elements of the sheet or the bundle may be either dependent or not identically distributed. In these cases the asymptotic behaviour of the overall sheet or bundle remains essentially the same as if the strength of all the elements were independent and identically distributed. In this case, the three limiting asymptotic distributions would form a good approximation. Galambos also emphasizes that assumptions of independence or identical distribution are difficult if not impossible to check in practice. There are cases where approximation by independent and identically distributed variables are valid. There are other cases where approximation is not valid.

It appears that the results outlined by Galambos have not been thoroughly applied to the asymptotic behaviour of extreme waves; although, a concern that large waves during storms are not independent and identically distributed has been raised in the literature: Challenor and Carter, (1981) Borgman and Resio, (1981) and Resio (1978). Whether or not the assumption of independence and identical distribution forms a valid approximation in this case is not known.

Challenor and Carter (1981) differentiate between different distributions of storm waves by considering only monthly maxima of storm waves. Their assumption is that within a month the large waves all come from the same initial distribution. Resio (1974) has shown that there may be twelve separate populations of extratropical storms off the east coast of the United States. The twelve storm types were differentiated by their storm track and their origin. There appears to be no correlation between storm track and months, due to the wide variation in atmospheric and air mass systems which determine the storm tracks and therefore the assumption of Challenor and Carter discussed above does not seem to be a rigid and general assumption.

The concept of defining storms as events having waves above a given threshold, first used by Petraukaus and Aagaard (1971) was also partly determined by the need for independent and identically distributed variables. The objective was to separate storm waves from more persistent wave conditions which have a different distribution. First used on the west coast, this approach was most likely an attempt to remove the effect of the swell component. Using this approach would not eliminate the need to define storm populations according to their storm track or their origin.

Resio (1978) showed that when a sequence of observations of significant wave heights occurring off Cape Hattaras, with a height above approximately 2 meters, were separated by storm type, the results for a 100 yr return period were 1.0 m lower than when all storm tracks were grouped together.

It is clear that there is no single test that is sufficient to determine the correctness of the assumption regarding identical

distributions. It is essential that this assumption be thoroughly evaluated prior to undertaking an extreme value analysis of waves for the Canadian Atlantic Continental Shelf.

In summary, to quote Galambos (1978), p 197, "Even if the approximation by independent and identically distributed variables is good, the credibility of a solution should be questioned if it starts by assuming that the variables are independent and identically distributed when they are not."

5.3 Selection of the Plotting Position

The only method by which a choice of probability distributions can be made is to plot the ordered sequence of recorded or hindcast data on suitable probability paper. The cumulative frequency assigned to each wave height in the sequence is calculated using one of several types of plotting positions. The general form of the plotting position is given as:

$$Fx(x) = \left[(k - a) \,/\, (N + b) \right] \qquad (5.4)$$

k = 1 for the lowest value and N for the largest value
N = number of observations
Fx(x) = observed probability that the value $X \leqslant x$

The parameters a and b are given below for several types of distributions:

Double Exponential (Gumbel, 1958)	a = 0 , b = 1	
Double Exponential (Gringorten, 1963)	a = 0.44, b = 0.12	
Weibull Distribution	a = 0.30 + 0.18 / C	$N > 20$
(Petrauskas and Aagaard, 1971)	b = 0.21 + 0.32 / C	

where C = exponent in the Weibull distribution

The values of a and b specified by Gringorten (1963) and Petrauskas and Aagaard (1971) are calculated to minimize the error between the exact plotting position, given by Blom (1958) and the approximated plotting positions given by Equation 5.4. Borgman and Resio (1981) suggest that the Gumbel plotting position, which is also the theoretical average plotting position, is as valid as any other plotting position and suggest it be used for all distributions.

There is no one correct plotting position and a subjective and arbitrary choice must be made. Several plotting positions will usually fit the data equally well so that a goodness of fit test cannot be made.

The difference which can result in an estimate of a given return period wave from using several plotting positions is generally small. An example undertaken for this study showed that for 8 years of wave data recorded at Logy Bay Newfoundland, the difference between the 100 year return period significant wave height calculated using the Gumbel plotting position and the Gringorten plotting position was 0.3 m. The data was extrapolated using the double exponential distribution and a line of best fit estimated by linear regression. The difference between the two plotting positions is small but a comparison of the plotted distributions, Figure 2, shows that there is considerable difference in the behaviour of the right hand tail of the distributions. This uncertainty must be considered in the acceptance or rejection of a distribution given its already subjective nature. Distributions should be plotted for several plotting positions.

5.4 Estimation of Parameters

The estimation of parameters of a distribution can be made by one of the following methods:

1) linear regression

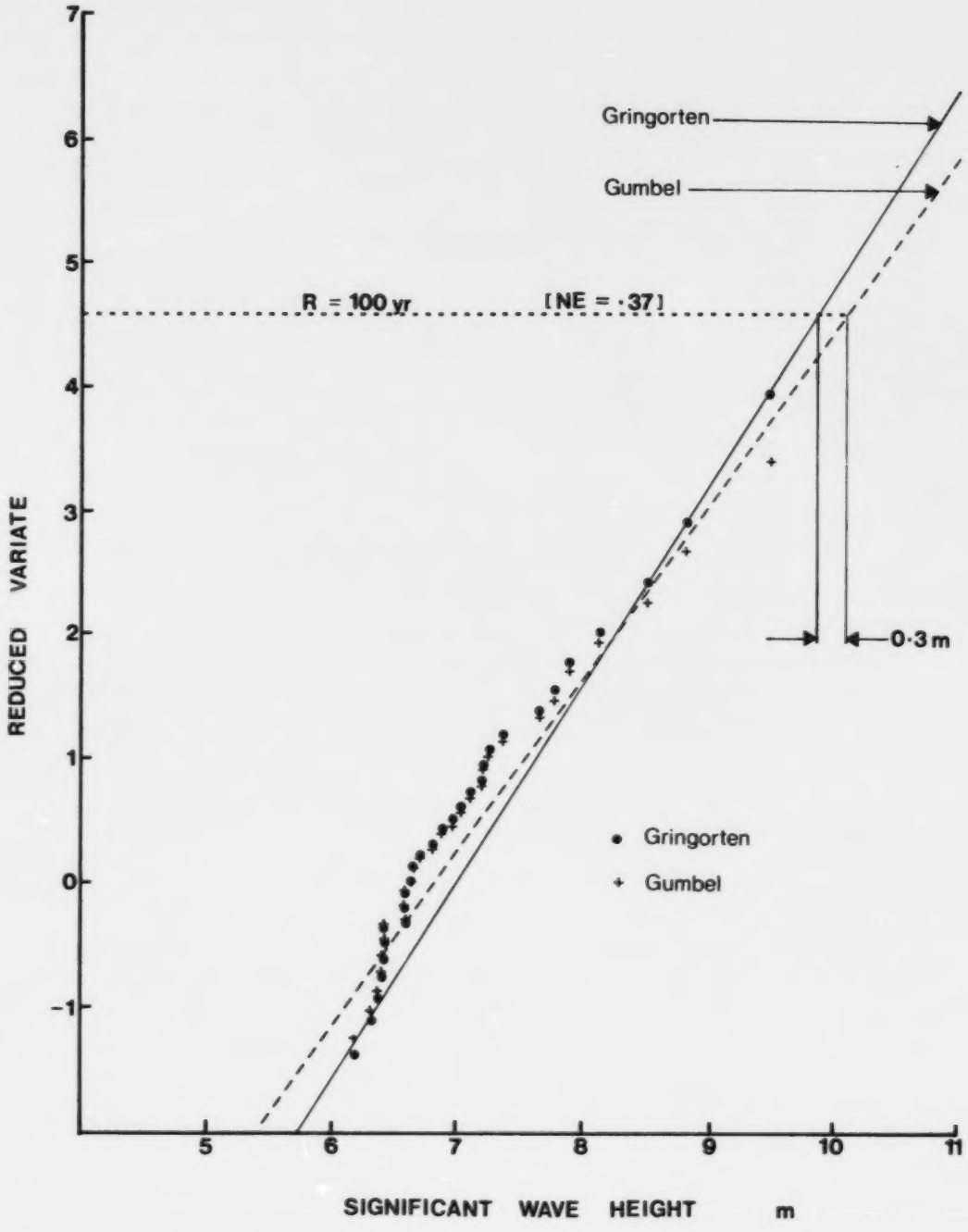
2) method of moments

3) method of maximum likelihood

The method of linear regression fits a straight line to the observed values of a variable (wave height) and the natural logarithms of the observed frequency or the reduced variate. Unbiased estimates of the parameters are obtained if corrections for the sample size are made. Linear regression has the disadvantage that confidence intervals calculated for the regression line implicitly assume that the initial distribution is correct. This assumption may not be true.

In the method of moments, parameters are estimated from the moments of the probability density function for the distribution. The moments are estimated from the sample and are biased estimates unless corrected for the sample size. The method of moments has the same disadvantages in calculating confidence intervals as does the linear regression method. For some distributions, moments do not exist and the method of moments cannot be used.

In the method of maximum likelihood, the parameters of the distribution are estimated so that the likelihood that the sample was obtained from the initial distribution is a maximum. The method requires that the likelihood function be differentiated with respect to each parameter which must be estimated and the resulting differential equations solved for their maxima. Analytical solutions are available for some distributions but in



PLOTTING POSITIONS

FIGURE 2

general the equations must be solved numerically. Details of the maximum likelihood method including a definition of the likelihood function are available in Kimball (1946), Gumbel (1958), Edwards (1972), Lawless (1974), and Griffiths (1980). Application of the maximum likelihood method for estimates of extreme waves using the Gumbel and Weibull distributions is given in Carter and Challenor (1978), Challenor (1979), and Challenor (1980).

Maximum likelihood estimates may be biased for small sample sizes but this bias can be removed. The method of maximum likelihood is generally the most preferable method of estimating parameters and has the advantage that realistic confidence intervals can be calculated and that no plotting position is required to estimate the parameters.

5.5 Uncertainty and Confidence Limits

There are two concepts of uncertainty which must be considered in the estimation of extreme wave statistics:

i) The uncertainty which results from estimating the true population parameters of a distribution from a sample of the population.

ii) The uncertainty associated with the risk of a certain event occurring.

In (i), confidence limits can be established which describe the probability that the true population lies within the limits estimated from the sample. These limits do not describe the probability of a specific event (at a defined return period) occurring. The event either occurs or it does not occur. The confidence limits describe the probability that to a given level of confidence (for example 68 percent) the true population distribution lies within the confidence limits calculated using parameters estimated from the sample. The extent of the range of uncertainty in this case is very sensitive to the number of samples on which an estimate of some extreme event is based. Increasing the number of samples decreases the range of uncertainty for up to approximately 50 to 80 samples. Beyond this, the uncertainty is relatively insensitive to an increasing number of samples.

In (ii), confidence limits can be established which quantify the risk of the event occurring. For example, if the distribution of the median (NE(x) = 0.5) wave height is estimated for some asymptotic distribution then the interquartile range for a given return period can be determined by substituting into equation 3.3 the values NE(x) = 0.75 (the value for which there is a 25 percent chance of a higher value) and NE(x) = 0.25 (the value for which there is a 25 percent chance of a lower value). In a similar manner the interdecile range can be calculated by

substituting the non-encounter probabilities of NE(x) = 0.9 and NE(x) = 0.1 into equation 3.3 and so on. Confidence limits for defined levels of risk can then be plotted on probability paper for any assumed asymptotic distribution by repeating the calculation at any number of return periods and drawing a curve through values of a constant non-encounter probability.

Methods for estimating uncertainty for case (i) are available in the literature included in the bibliography and in the list of references. Petrauskas and Aagaard (1971) discuss the problems of estimating the uncertainty for case (i) and present a Monte Carlo simulation experiment which can be used to estimate the uncertainty of not knowing the true population. Challenor (1979 and 1980) presents graphs from which the uncertainty for the double exponential and Weibull distributions can be estimated. Their graphs are based on a Monte Carlo experiment which is similar to that of Petrauskas and Aagaard (1971). The method of defining uncertainty for case (ii) is discussed in more detail in Borgman and Resio (1977) and Borgman and Resio (1981).

Uncertainty in the estimate of extreme wave statistics which is introduced by practical considerations such as the quality and extent of recorded data, the location of the data, the method of calculation (for a wave hindcast), or the effect of climate are discussed in Chapter 6.

6 LIMITATIONS OF PROCEDURES TO DETERMINE EXTREME WAVE STATISTICS

6.1 Limitations of Recorded Data

Recorded Wave Data

When recorded wave data is extrapolated to estimate extreme wave statistics, much of the resulting uncertainty is due to the data itself. Limitations of recorded wave data which contribute to this uncertainty are:

a) The length of coverage provided by the wave data

b) Characteristics of the data which are particular to the recording site and are not transferrable elsewhere

c) Meteorological conditions during the period of recordings.

There is a general rule of thumb that the length of coverage of recorded data should be equal to at least one half the return period for the project, Borgman and Resio (1981). The reliability of an extrapolation beyond twice the length of the original data should be viewed with caution because the range of confidence limits describing the reliability of the estimate increases very quickly after this point.

Wave data have been recorded in the waters of the Canadian Atlantic Continental Shelf only since 1972 and, with the exception of data recorded at Logy Bay, Newfoundland, for only one or two years at a time at any one site. Reliable extrapolations for long return periods would not be possible with this data.

Recorded wave data are seldom available for the specific site of interest. The recorded wave data often contains particular characteristics which are unique to the recording site. These characteristics may include: depth limitations, fetch limitations, refraction (depth or current), diffraction, and specific storm tracks not characteristic of the site of interest.

Variability in the sample may occur if the interval during which coverage is obtained is an exceptionally stormy or calm interval. Short periods of recording are especially prone to sample variability of this type. An example is illustrated in Table 2 where wave data recorded offshore of Logy Bay, Newfoundland, during the ice free seasons between 1972 and 1979 were extrapolated to determine the 20, 50, and 100 year return period significant wave heights. The eight years of recorded wave data were subdivided into five subsets of four years each. The range of estimated wave heights for each subset are given on Table 2. Depending on which subset of wave data is used the estimate of the extreme wave height lies within a range of 2.2, 2.6, and 2.8

meters for the 20, 50, and 100 year return periods respectively.

Table 2

Extreme Significant Wave Heights for Logy Bay

Return Period (yr)	Significant Wave Height (m) Subset 1	2	3	4	5	Maximum Difference in height (m)
20	9.6	10.9	11.3	11.1	11.8	2.2
50	10.2	11.8	12.2	12.0	12.8	2.6
100	10.8	12.5	12.9	12.7	13.6	2.8
No. Storms	30	31	46	47	31	

Note: 1) Storm defined as Hs greater or equal to 4.6 m.
2) Extrapolation made using double exponential distribution.
3) Data grouped into subsets as follows:

Subset	Interval
1	1972 - 1975
2	1973 - 1976
3	1974 - 1977
4	1975 - 1978
5	1978 - 1979

The estimate for the 20, 50, and 100 year return period significant wave heights using all eight years of data are 10.5, 11.4, and 12.0 m respectively. These values are average values within each range. It must be noted that the wave heights based on the eight years of data are no more correct than the values based on 4 years. Both sets of estimates have been extrapolated several times beyond the length of the original recordings and are most likely unreliable estimates for such long return periods. The example does illustrate the variability in an estimate which can result from short periods of coverage of recorded data.

Much of the variability of the above example is due to the number of storms which occur in each sample. The number of storms are listed in Table 2 for each subset. In Figure 3 the value of the highest significant wave height in each storm is shown and it can be seen how the storms which occurred during a particular interval biased the estimates. The largest estimate comes from subset 5 which contains the three largest values. The smallest estimate comes from subset 1 which includes only the smaller values. The relationship is clearly more complex than suggested by the above discussion, but the example does serve to illustrate some of the problems of using short lengths of recordings to extrapolate to long return periods.

These differences in the estimate of the extreme significant wave

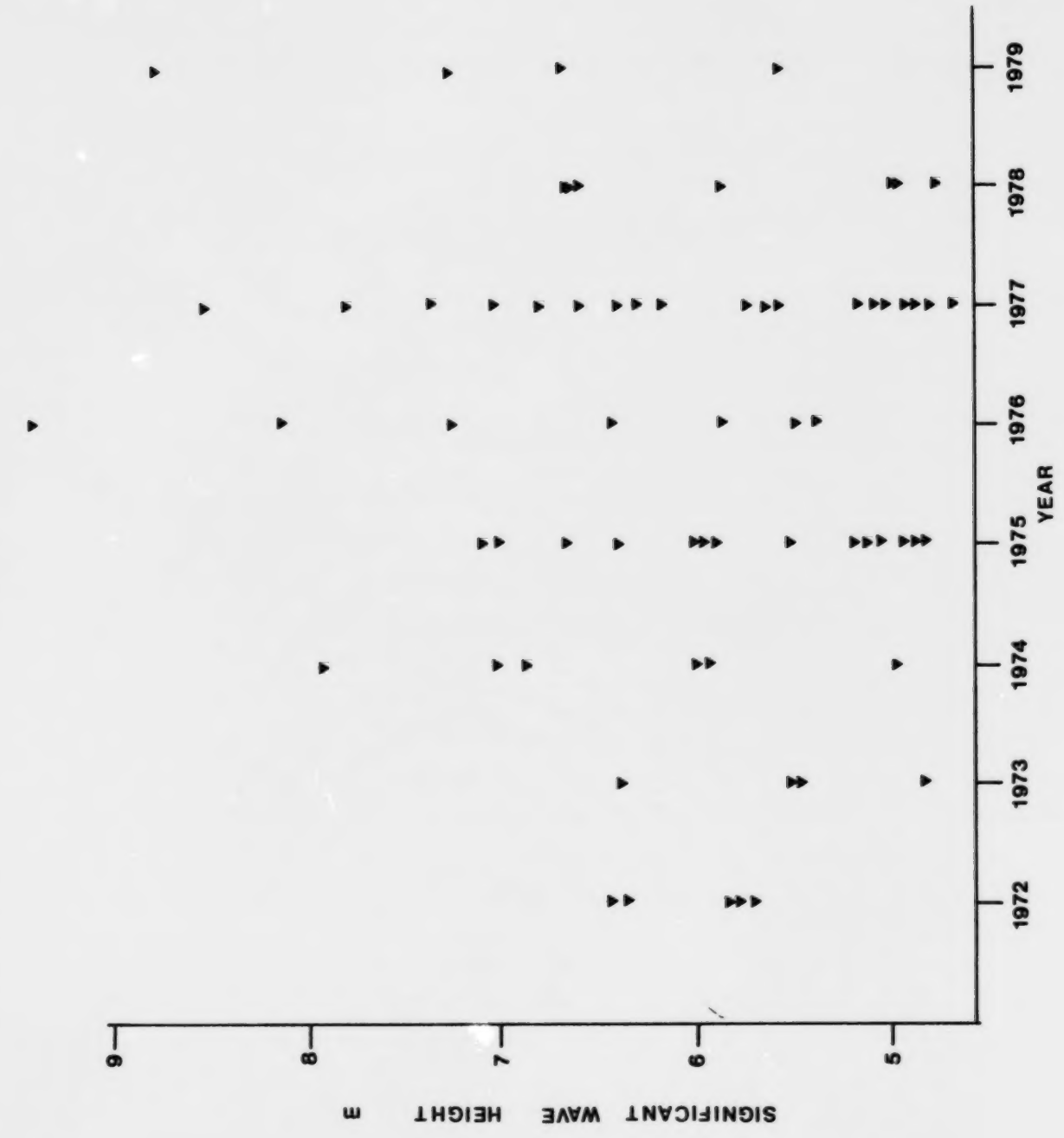

SIGNIFICANT WAVE HEIGHT m
9
8
7
6
5
1972
1973
1974
1975
1976
1977
1978
1979
YEAR

FIGURE 3

height for a given return period can be attributed to cyclical variation in the intensity of storms or calm periods in the North Atlantic. Resio and Hayden (1975) have identified a cyclical variation in large scale circulation patterns in the atmosphere of between 3 and 5 years in duration. In addition to this cyclical variation, Resio and Hayden identified a long term, or secular variation in the large scale circulation patterns which may produce increasingly stormy years in the future. This aspect of long term variations is discussed in further detail in Section 6.5.

Given the variation in extreme significant wave heights which can occur using data from Logy Bay as well as the limited coverage elsewhere on the Canadian Atlantic Continental Shelf it is unlikely that reliable extreme value estimates can be expected from the extrapolation of recorded wave data.

Recorded Meteorological Data

The hindcasting and extrapolation of storm wave data depends for its accuracy on the reliability and accuracy of the temporal and spatial definition of the overwater wind field. The accuracy of the overwater wind field depends on the extent and quality of recorded meteorological data. These recorded meteorological data will form an upper bound on the reliability of any estimate of extreme wave statistics.

Early historical meteorological data are most likely to be limited in accuracy due to the methods and density of observations. Furthermore, meteorological data from areas of the ocean far removed from land or major shipping lanes will also be limited in accuracy.

The majority of meteorological conditions over the world's oceans comes from routine ship observations and a limited number of ocean stations such as the Canadian weathership station Papa in the North Pacific. These data are limited in accuracy by the nature of the type and quality of observations on the ship, and recent studies, Lazanoff and Stevenson, (1977), Cardone, et al, (1980), have found that winds during storms which were estimated from this collected data tend to be underestimated by up to 10 to 15 percent. In areas where meteorological data are available from exploratory and production drilling rigs, the quality of recorded data may be significantly improved.

Resio and Vincent (1979), describe the upgrading of meteorological data which the Waterways Experiment Station undertook in the development of a wave climatology of the North Atlantic. During this process, synoptic weather data for storms was blended with ship observations of wind speed with the synoptic data taking precedence. Estimates of wind speeds at a 19.5 m elevation had an rms error of 2.5 m/s when compared to winds recorded at Sable Island after this blending process.

The specification of the overwater wind field is also limited by the current state of knowledge of the marine boundary layer and the planetary boundary layer in which the major circulation features which control the winds are located.

Experience with recent hindcast studies, Ewing, (1979) suggests that overwater winds derived from synoptic scale meteorological measurements are accurate to within 2 to 3 m/s. Cardone, et al, (1978) compared three procedures for determining overwater wind fields and found that overwater wind fields estimated from surface measurements of pressure or wind were limited to an accuracy of plus or minus 2.5 m/s and plus or minus 20 degrees in direction. Furthermore, the winds were biased and tended to be underestimated during storms.

Any study to determine extreme wave values must consider the accuracy of the overwater wind fields used to hindcast storms and wherever possible, derived winds should be compared with recorded winds to determine if any bias in the wind speed exists during storms.

6.2 Physical Accuracy of a Hindcast Model

The uncertainty associated with an estimate of extreme wave statistics based on hindcast wave data will also be limited by the accuracy with which the hindcast procedure represents the wave generation process. A complete discussion of parametric or spectral hindcast models is beyond the scope of this report, but the extent of accuracy which can be expected based on comparisons of hindcast results and recorded data will be discussed.

In a study of both parametric and spectral hindcast models, Ewing, (1979) found that the standard derivation of hindcast significant wave heights when compared to recorded data was between 1.4 and 0.2 m with the physically most realistic models giving better estimates than procedures based primarily on empirical relationships from recorded data. The standard deviation increased with wave height and an average deviation of 1.0 m could be expected for large (up to 14 m) significant wave heights.

In a recent study which compared archived hindcast wave data with wave data recorded with Datawell wave rider buoys located in the proximity of exploratory drilling rigs on the Grand Banks and the Nova Scotia Continental Shelf, Baird and Readshaw (1981), found that it was not possible to conclude if the hindcast data provided an accurate representation of the wave climate on the Canadian Atlantic Continental Shelf. In particular, the comparison of hindcast and recorded wave heights at the peak of a storm showed a much greater variation than plus or minus 1.0 m. The increased scatter was partly due to the fact that archived hindcast wave data was only available at a limited number of grid points, most of which were 82 to 380 km from a wave recorder.

It is essential that any hindcast wave data which are to be used to estimate extreme wave values be verified and that a quantitative estimate of any uncertainty due to the hindcasting procedure be included in the total uncertainty of the extreme wave estimate.

6.3 Definition of a Storm

The majority of extreme value analysis procedures which are based on hindcast wave data use the maximum significant heights associated with individual storms. This type of analysis requires an objective definition of a storm. Ideally, this definition would exclude storms which are not independent: for instance, storms which originate in the same large scale weather disturbance. Furthermore it would include only those storms which could produce extreme wave events. Such an objective definition does not exist.

The current practice is to define a storm as an independent event producing waves above a predetermined threshold condition. The threshold is expressed as a specific significant wave height. Independence is ensured by only hindcasting storm waves for selected storms remote in time from each other. When recorded wave data are used or if a continuous hindcast is undertaken, the determination of independent storm events is even more difficult as two maxima, both greater than the threshold condition, may originate from the same system. In some cases an arbitrary spacing of 24 or 36 hours is specified to ensure independence. This is not a satisfactory procedure as extratropical storms can stall over the Canadian Atlantic Continental Shelf for long periods of time (5 to 7 days) due to blocking of the low pressure system by an Atlantic High pressure system. The only satisfactory procedure appears to be to refer to synoptic charts to determine if sequential storms are independent.

The choice of the threshold is a subjective choice and may have a significant effect on the estimated extreme values. An example of the influence of the choice of threshold has been prepared for this study using storm waves selected from recorded wave data for Logy Bay, Newfoundland, for thresholds of 4.6 m and 6.1 m. The results are summarized in Table 3 and show that depending on the threshold chosen, the estimate of extreme waves varies by 0.73 m for the 100 year return period.

Table 3

The Effect of the Threshold Condition

Return Period (yr)	Extreme Significant Wave Height (m) Threshold		Difference in Estimate (m)
	Hs > 4.6 m	Hs > 6.1	
20	10.5	10.1	0.4
50	11.4	10.7	0.7
100	12.0	11.2	0.8

Note: 1) Based on recorded wave data from Logy Bay, Newfoundland.
2) Extrapolation by the double exponential distribution.

Haring, (1979) and Ewing, Worthington and Weare (1979) both estimated extreme wave statistics using identical results of the NORSWAM numerical wave hindcast study. Haring used a threshold of 6 m to define storms, Ewing used a threshold of 5 m. Although their results are not for the same grid points in the NORSWAM grid, but are averages over similar or nearby areas, the estimates differed as shown in Table 4 for the 100 year return period wave:

Table 4

Grid Point Ewing et al (1979)	Threshold >5m	Grid Area Haring (1979)	Threshold >6m
Famita	16.2	Area A	13.4 - 14.3
Stevenson	16.2	Area D	12.5 - 13.7
Fitzroy	18.9	Area E	12.5 - 13.4

Not all the difference can be attributed to the threshold criteria as each study used a slightly different procedure to average the results for specific grid points over a larger area. Spatial representation of hindcast results is discussed in Section 6.4.

In another example, 20 years of continuous wave data hindcast over the Canadian Atlantic Continental Shelf by the Waterways Experiment Station, U. S. Army Corps of Engineers were used. The data were hindcast for position 49 degrees, 38 minutes N, 50 degrees, 45 minutes W, and the 100 year return period significant wave height was 14.2 m, 14.8 m, and 15.9 m for storm threshold conditions defined as >5.0 m, >7.0 m, and >9.0 m respectively.

There is no consistent pattern to the variation of a given return period significant wave height resulting from different threshold

conitions. Using recorded data from Logy Bay, Newfoundland, the estimated 100 year significant wave height decreased as the threshold increased. The 100 year signficant wave height estimated from the hindcast data increased as the threshold increased. If storms are to be defined by a threshold condition, further investigation is required. This investigation should include the development of a statistical understanding of the effect of the threshold value on the estimation of extreme values. A discussion of two similar concepts in statistics known as censorship and truncation is available in Johnson and Kotz (1970). The "peak over threshold" approach to predicting annual flood events is also discussed in NERC (1975).

6.4 Spatial Resolution of Hindcast Storms

The distribution and relative magnitude of hindcast wave parameters will vary at individual grid points in the hindcast model. The extent of the difference will depend on the track of the storm centre, the size of the storm and the spacing of the hindcast grid. If a specific location of interest is some distance from one of the grid points, then an averaging, weighting, or interpolation scheme will be required.

One procedure which has been used (Haring, 1979) is to average the results for all grid points in and around the area of interest. Comparison of the results of Ewing, Worthington and Weare (1979) who did not average any results and Haring (1979) for the same area of the North Sea shows, Table 4, that differences of up to 6.5 m may result when such a procedure is used. Some of this difference may be due to their difference in the definition of the threshold condition discussed above, but it is not possible to isolate the contributions of the threshold and the averaging process.

This difference in estimated extreme wave heights which can result when waves are hindcast at adjacent grid points is most likely due to the difference in the intensity and number of storm tracks which pass over the grid points and will be sensitive to the resolution of the grid. Small-grid spacings will provide a more accurate definition of the wind field and should result in smaller variations. A large grid spacing may result in entire storms, especially those of tropical origin being missed in the hindcast results.

An alternate procedure, used by Ward, Evans and Pompa (1978) is to hindcast waves for a given storm at a single location. The location is chosen so that the maximum wave conditions generated by the storm occur at that point. In an application of this procedure for the continental shelf east of Nova Scotia, 23 severe storms were selected and used to hindcast the storm significant wave heights. The exact details of the procedure are contained in a proprietary report and are not available. However, it seems likely that this procedure leads to

conservative results as it assumes that the storm tracks of all the storms pass immediately over the area of interest. Resio and Hayden (1975) provide an extensive discussion of the statistics of storm tracks of different types. Storm tracks are discussed further in Chapter 7.

The method used by Ward, Evans, and Pompa (1978) may have application off the east coast of Canada for storms of tropical origin (hurricanes and tropical storms) whose storm tracks in the northern latitudes are influenced to a large extent by large scale atmospheric disturbances in the subtropical and northern latitudes. It is probable that the number of hurricanes whose tracks extend over the Canadian Atlantic Continental Shelf is too small to produce reliable statistics. Therefore, it may be necessary to develop extreme wave statistics for these storms based on the worst wave conditions which each hurricane or tropical storm could develop in the area of interest, ignoring the track which was taken by the actual storm. Another procedure which incorporates the statistics of the synoptic coupling between individual storm tracks and large scale weather systems in the northern hemisphere is discussed in Chapter 8.

In summary, there is no objective procedure for accounting for the spatial effects of a finite grid size in the hindcast procedure.

An averaging procedure such as used by Haring (1979) is probably not a satisfactory procedure to produce extreme wave statistics because of the information which is lost during the averaging procedure. A more acceptable procedure would be to interpolate amongst adjacent grid points but the acceptability of this procedure will depend on the grid spacing which is used in the hindcasting procedure.

The procedure which should be undertaken can only be evaluated on a site specific basis and only after extreme value analyses have been completed at all adjacent grid points for a specific area of interest.

6.5 The Influence of Climate on Extreme Wave Statistics

Climate has an influence on the estimation of extreme wave statistics. This influence may be due to mixed populations of storm types, short term cyclical changes in the frequency of storms, or long term secular changes in the climate.

Mixed populations of storms result when storms of either tropical or extratropical origin occur in the same region. This mix of storm types occurs over the Canadian Atlantic Continental Shelf. Hurricanes which are in the process of recurving northeastward towards the North Atlantic or synoptic scale low pressure systems moving eastward along the Atlantic Polar Front may both produce large storm waves.

Resio (1978) has suggested that depending on the track of extratropical storms along the Eastern Seaboard of the United States, up to 12 separate populations of storm type can exist off the North Carolina coastline. Resio determined that the distribution of extremes of all types of storm generated waves (defined as greater than 2 m) determined from a Fisher Tippett Type II asymptotic distribution gave a 100 year return period of 11.6 m. When the storm waves were stratified by storm type and fit by a Type I asymptotic distribution the 100 year return period decreased to 10.5 m.

The plot of the unstratified data showed some curvature to it while the data stratified by storm type had no curvature. Resio (1978) concluded that curvature in the distribution of extreme wave data is the direct consequence of unstratified storm types or mixed populations. Only by fitting several asymptotic distributions to the wave data and plotting the resulting fitted distribution and data can such curvature and possible dependency on storm type be identified.

A similar case of curvature in the plotted data and the presence of a mixed population of storm types was found by Gourlay (1974) in an examination of wave data off Mangalore Harbour on the west coast of India.

The existence of mixed populations of tropical and extratropical storms over the Canadian Atlantic Continental Shelf has been demonstrated by Ward, Evans, and Pompa (1978) who determined extreme wave statistics for hurricanes and extratropical storms over the Georges Bank and the Scotian Shelf. In a recent comparison of hindcast and recorded wave data between January 1973 and December 1975 from the continental shelf off the east coast of Canada, Baird and Readshaw (1981), it was found that for the two most intense storms (producing peak significant waves of equal height), one was of extratropical origin while the other was of tropical origin. The meteorological processes from which hurricanes and extratropical storms develop are not the same (the processes are discussed in Chapter 7) and it is therefore necessary to separate storms from these two different populations.

In an analysis of extreme wave statistics for the Canadian Atlantic Continental Shelf, the plotted distributions should be carefully examined to determine if stratification by storm type for extratropical storms will be necessary. The origin of storms which are present over the Canadian Atlantic Continental Shelf and their characteristic tracks are discussed in Chapter 7.

In a study of large scale and synoptic scale weather patterns in the Northern Hemisphere, Resio (1978) found that short term cyclical changes in weather patterns of 3 to 5 years duration produce cycles of stormy and calm years. In the example in Section 6.1, recorded significant wave heights for Logy Bay, Newfoundland, were shown to vary considerably over a 3 to 4 year

period. A second example using hindcast wave data from the U. S. Army Corps of Engineers has also been prepared. Wave data for the period 1971 to 1975, inclusive and 1956 to 1975 inclusive was used to calculate extreme wave statistics for the same hindcast grid station. As the example in Section 6.3, the range of significant wave heights for the three threshold conditions is shown in Table 5.

Table 5

	Threshold Condition		
	>5 m	>7 m	>9 m
Data from 1971 - 1975	16.9 m	17.8 m	20.5 m
Data from 1956 - 1975	14.2 m	14.8 m	15.9 m

The effect of short term cycles in the climate can not be totally eliminated in an extreme wave statistics analysis. Resio (1978) has described a procedure for estimating extreme waves which incorporates the probability of short term cyclical changes in the climate. The procedure is discussed further in Chapter 8.

7 THE CLIMATOLOGY OF THE CANADIAN ATLANTIC CONTINENTAL SHELF

The wave climate of the Canadian Atlantic Continental Shelf, which includes the Nova Scotia Shelf, the Grand Banks of Newfoundland and the Labrador Shelf is produced by storms of both extratropical and tropical origin.

Extratropical cyclones have their origin in the frontal systems developed by the interaction of cold and warm air masses in eastern North America. These air masses are the polar continental air mass and the maritime tropical air mass. The boundary between the two is ideally described as a wavy surface where typically a low pressure centre develops in the apex of a wave. Warm and cold fronts develop around the low pressure centre as the warm air mass is enclosed by cold air. The system matures as it is moved eastward by the upper atmosphere jet stream until the warm front occludes and the system is reabsorbed by the cold air mass.

During this process the low pressure centre may intensify and the pressure gradient between the centre of low pressure and a nearby high pressure system may produce storm force winds.

The physics of this system are complex and an explanation is not provided here. However, many of the storms which generate severe waves over the Atlantic Continental Shelf are of this origin, especially during the late fall, winter and early spring months.

Tropical cyclones have their origin in the tropical latitudes and are produced from a similar wave-like phenomenon. North Atlantic tropical cyclones classically have their origin off the coast of Africa near 15 degrees North. In addition some tropical cyclones have their origin in the remnants of old polar troughs or upper atmosphere low pressure centres. Another class of tropical cyclones exhibit both tropical and extratropical characteristics and are defined as subtropical cyclones. These cyclones develop in subtropical waters as non-frontal low pressure systems and may attain hurricane characteristics.

Tropical cyclones are considered to be a major factor in maintaining the atmospheric heat and moisture balance between the tropics and more northern latitudes. These cyclones are thought to act as a type of safety valve, limiting the continuing build-up of heat and energy in the tropical areas. A tropical cyclone is characterized by an "eye", an area of low wind speeds, low pressure and clear skies. At the perimeter of the eye, the wind speeds in the counter clockwise circulation system reach a maximum speed. A tropical cyclone does not have associated with it a system of warm and cold fronts as found in an extratropical storm.

Many tropical or subtropical cyclones develop the characteristics of extratropical cyclones after moving out of the tropics. During this downgrading process, the circulation of the cyclone

expands and the maximum wind speed decreases. In addition, the forward speed of the cyclone increases and the circulation and distribution of winds, temperature and rainfall around the centre become increasingly assymetric. In some cases, small features of a tropical cyclone such as an eye and a small area of hurricane force winds near the centre may be retained within the larger extratropical system. In general, windspeeds subside as a tropical cyclone attains extratropical characteristics although reintensification of the cyclone may occur if extratropical mechanisms offset the loss of the tropical energy source.

Tropical and subtropical cyclones are classified by the National Hurricane Centre, National Oceanic and Atmospheric Administration (NOAA), United States in the following manner:

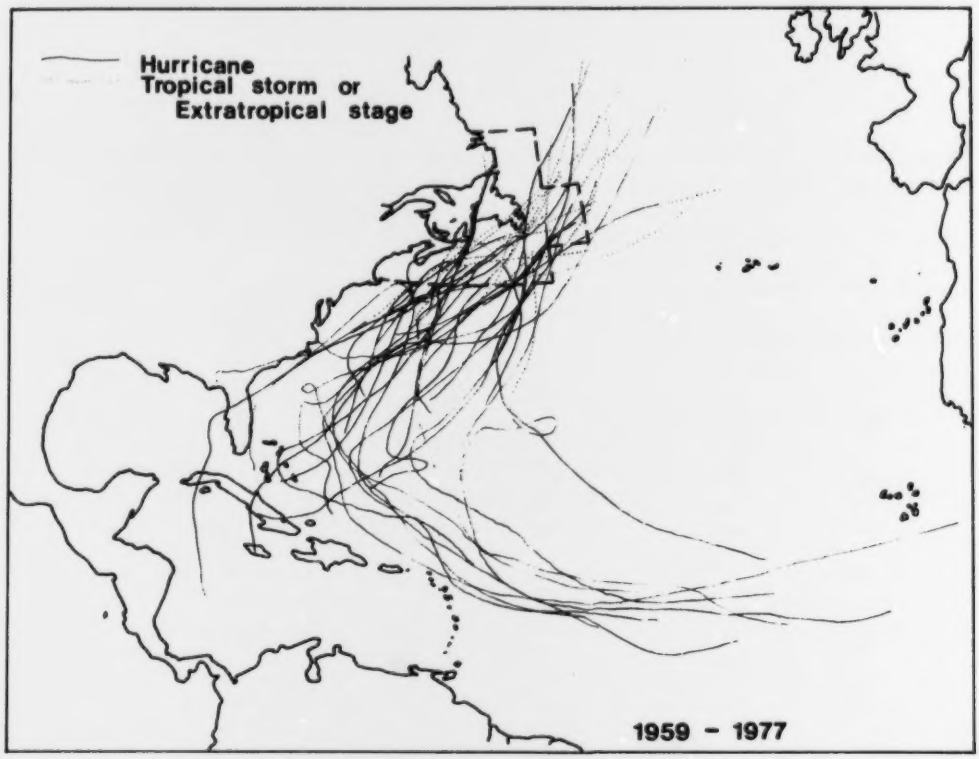
	Wind Speed
i) Tropical or subtropical depression	$<$61 kph
ii) Tropical or subtropical storm	$>$61 kph and $<$ 118 kph
iii) Hurricane	$>$118 kph

There is no equivalent classification for extratropical cyclones.

During the perid 1944 to 1979, 81 tropical cyclones crossed the Canadian Atlantic Continental Shelf. The majority crossed the shelf and moved into the North Atlantic south of 50 degrees N although some storms did continue northward into Davis Strait. The tracks of these cyclones are shown in Figure 4. The shaded area includes the Canadian Atlantic Continental Shelf. Of these 81 cyclones, 33 were classified as hurricanes as they passed latitude 43 degrees N. The remainder were classified as either tropical storms or tropical cyclones downgraded into their extratropical stage. Many of the 33 hurricanes were also downgraded to either tropical storms or extratropical stage after they crossed the 43 degrees N latitude. In some cases the storms reintensified and continued northeast as hurricanes, passing over the Grand Banks area and occasionally north to the Labrador Shelf.

North America derives its weather from 3 principal sources, the Pacific Ocean, the Arctic and the Atlantic Ocean and the Gulf of Mexico. Storms originate in all three sources and almost all of the continent is subjected to storms from all three sources at one time or another.

The two air masses which dominate the eastern half of the continent are the continental polar air mass, originating in the Northwest Territories and the Arctic archipelego, and the maritime tropical air mass from the Atlantic Ocean and Gulf of Mexico. These two airmasses meet at the Atlantic Polar Front which forms the major frontal zone in eastern North America. It is along this frontal zone that most of the extratropical storms over the Canadian Atlantic Continental Shelf originate. The position of the Atlantic Polar Front varies considerably during


Hurricane
Tropical storm or
Extratropical stage
1959 - 1977

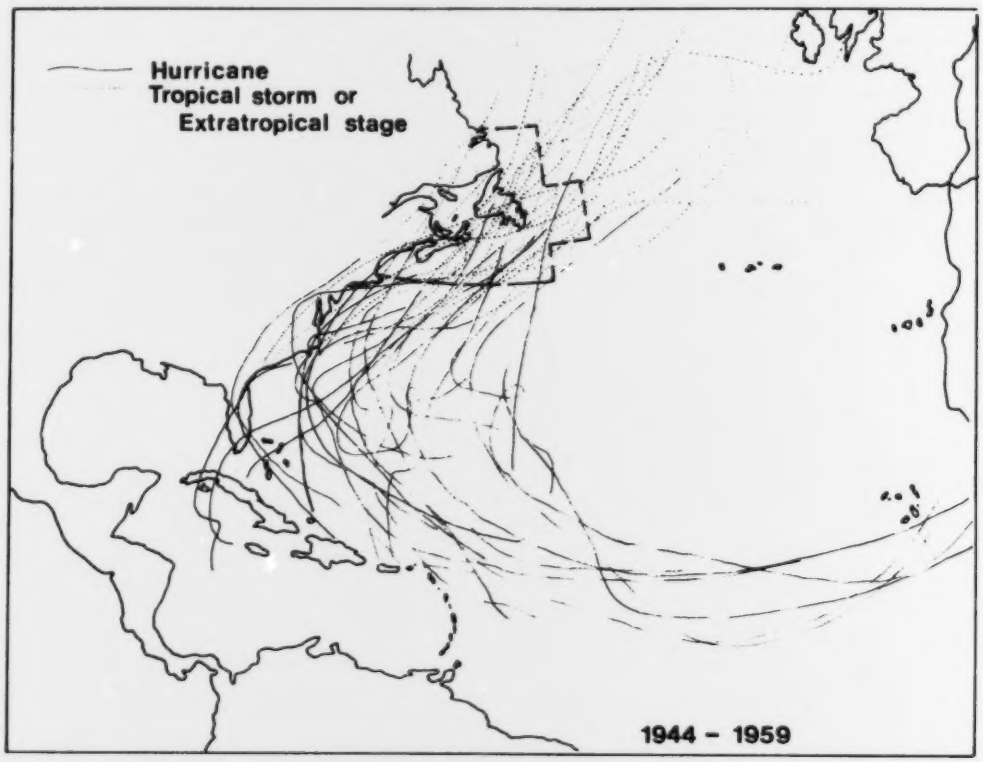

Hurricane
Tropical storm or
Extratropical stage
1944 - 1959

FIGURE 4

the year. In the winter months, the front lies along the east coast of North America and intense extratropical storms are formed in the Cape Hattaras to Cape Cod area of the eastern United States Continental Shelf. Many of these storms make their way north to the Canadian Atlantic Continental Shelf area and form intense storms as their track takes them over open water. In the spring and summer months the Atlantic Polar Front lies further north extending from the midwest plains of the United States, northeastward across the Labrador coast.

On the west coast of North America the dominant air mass is the Pacific maritime tropical airmass. This mass is usually separated from the continental polar air mass by the western cordillera, but at times, the moist maritime air will cross the mountain ranges and cyclogenesis will occur in the foothills area. The storms which are produced by the formation of this Rocky Mountain Front are often very quick moving and intense storms. Storms of this type form both in the Alberta foothills and the Colorado foothills, although the storms which form in the Colorado area usually form along the Atlantic Polar Front and include Atlantic maritime tropical air. The Alberta depressions may or may not include Pacific maritime tropical air. The exact nature of the formation of these Alberta and Colorado storms is not well understood.

It is along the Atlantic Polar Front and the Rocky Mountain Front that many of the extratropical storms on the Canadian Atlantic Continental Shelf originate. The tracks vary from winter and summer, but it is from these origins which it is suggested that storm types should be identified for an extreme value analysis for the Canadian Atlantic Shelf.

Hare (1961) lists the major storm types in North America and their characteristics. Those types which result in extratropical storms over the Canadian Atlantic Continental Shelf are listed below with their characteristics.

Table 5

STORM TYPE	ORIGIN	FRONTS AND AIR MASSES INVOLVED	SEASON
Alberta	Rocky Mountain foothills north of 50 N	Rocky Mountain front. cP and mP air	All year (intense in the winter)
Colorado	Rocky Mountain foothills south of 50 N	Waves on the Atlantic Polar Front (thought to be triggered by mP air)	All year
Atlantic Coast	Gulf of Mexico Cape Hattaras to Cape Race Nfld	Waves on the Atlantic Polar Front cP and mT air	All year (most severe Atlantic storms are in this group during winter
Hurricanes and Tropical Storms	Caribbean and Tropical Atlantic	mT air, down-grades to blend with Atlantic Polar Front	Late summer and fall

This discussion is not exhaustive but is intended to identify the origins of storms which may contribute to storm waves on the Canadian Atlantic Continental Shelf. Any analysis of extreme waves in this area should consider the separate populations which may result from these storms.

8 PROJECT STORMS

In addition to the extrapolation of either recorded or hindcast wave data, a third approach, The Project Storm Approach, exists for the estimation of extreme wave statistics. The project storm approach involves developing a statistical model of the parameters which catagorize the development of a storm. Based on the statistical model, the required storm for a given return period can be estimated and the waves during the storm then hindcast using a suitable hindcast procedure. This approach has not yet been used for estimating extreme waves due to a limited knowledge of all the governing parameters which describe a storm. Resio (1978) has suggested a project storm approach should be used to formulate a more reliable estimate of future extreme events. Project storms have been used along the Eastern Seaboard of the United States and in the Gulf of Mexico to predict extreme storm surge levels. Two project storm types have been defined:

a) the Standard Project Hurricane: defined as a hypothetical hurricane that is intended to represent the most severe combination of hurricane parameters that is reasonably characteristic of a region, excluding rare combinations.

b) the Probable Maximum Hurricane: defined as a hypothetical hurricane having a combination of characteristic parameters which will result in the most severe storm that is reasonably possible in the region if the hurricane should approach the region along a critical path and at an optimum rate of movement.

The project storm is defined from a statistical understanding of the parameters which define the intensity of the hurricane. The project storm has application where little or no recorded data is available. This lack of recorded data is true of the Canadian Atlantic Continental Shelf region where a significant number of tropical cyclones have passed over the region with such widely varying storm tracks that both land based meteorological stations and wave recorders have measured wind speeds or wave conditions for only a small proportion of the total number of tropical cyclones.

The concept of a regional probable maximum storm is currently used in the field of hydrology for the estimation of extreme flood events in regions where recorded data is scarce or of limited duration. The World Meteorological Organization (1977) has published guidelines for estimating this storm event for extratropical storms and the procedure may have application over the Canadian Atlantic Continental Shelf.

Resio (1978) examined the probabilistic coupling between storm waves and large scale atmospheric changes and developed an expression (Equation 8, Resio, 1978) for the probability distribution of extreme waves which makes allowance for the joint

probability of large scale atmospheric changes and the resulting change in the frequency of storms along a given storm track. The procedure requires prior knowledge of large scale circulation patterns in the atmosphere if it is to be used to estimate extreme wave statistics and is therefore not of practical use without further development. Resio did develop a procedure to calculate the change in return period knowing or estimating the change in the frequency of a single storm type. The procedure, (summarized in Appendix 3) is only valid when extreme waves result from a single storm type, in which case the magnitude of change in the return period is of the same order as the sum of the changes of the probability of the storm type. In an example on the Eastern Seaboard, Resio found that when the frequency of a given storm type increased by 20 percent between 1920 and 1960, the return period of the 50 year event based on the decade of 1920 decreased to 41.6 years for the decade of 1960.

Resio stressed that short term cyclical variations in the climate can dramatically alter the occurance of rare storms and that the climatic conditions which prevailed during a post occurance of this particular storm type should be carefully reviewed.

Resio's results suggest that estimates of extreme wave statistics must include an analysis of the coupling between the frequency of storm types and large scale atmospheric conditions. Such an analysis would be expected to show that where short term cyclical changes in the frequency of storm types did occur, there is a real and quantifiable risk of a previous stormy cycle occurring again and producing severe storm waves. Under such circumstances, the return period assigned to a given significant wave height in the absence of such an analysis can be significantly overestimated.

9 DISCUSSION AND RECOMMENDATIONS

9.1 Discussion

This study has identified three procedures for estimating extreme significant wave heights. The first procedure which uses recorded wave data is unlikely to provide reliable estimates of extreme wave statistics. The second procedure which uses wave data hindcast during storms may be the most reliable procedure if a systematic evaluation of sources of uncertainty in the procedure is undertaken. The third procedure, Project Storms, may be applicable where insufficient information is available for the second procedure.

This review of the state of the art of estimating extreme wave statistics has revealed two important points concerning the second procedure. First, there is no single objective procedure for estimating extreme wave statistics. Every method has its own theoretical, statistical, or empirical justification and any one method may be as valid as another. Second, there can be considerable difference in the results from different methods even when both procedures use the same data. Furthermore, there is no single study which has systematically examined all the areas of uncertainty which are present in a procedure.

It is concluded, therefore, that a study to estimate reliable extreme value statistics on the Canadian Atlantic Continental Shelf must examine and quantify the uncertainty which may result from those topics discussed in this report. Uncertainty will result from the following:

a) The choice of asymptotic probability distribution.

b) Violation of the assumption of stationary, independent, and identically distributed variables.

c) The choice of the plotting position formula.

d) The method for estimating the parameters of the asymptotic probability distribution.

e) The definition of a storm and in particular of the threshold condition.

f) The spatial resolution of recorded or hindcast data.

g) Mixed populations of storm types.

h) The occurence of short term cyclical changes in the climate

i) The possibility of an intense storm occurring due to large circulation patterns in the atmosphere which had only

rarely occurred in the past.

j) The risk which can be accepted in the estimate of the extreme wave height.

Once the significant wave height for a given return period has been estimated it is necessary to determine:

a) the period associated with the significant wave height.

b) the predominant directions of the waves during the storm.

c) the maximum wave height during the extreme storm event

d) the period of the maximum wave height

e) the crest elevation of the significant or maximum wave height

f) the spectral shape of the sea state during the extreme storm.

This list is not exhaustive and other parameters may be required depending on the design requirements of a particular structure. Estimating these parameters is not straight forward and procedures to do this have not been reviewed during this study. Further studies should be undertaken to review these procedures and make recommendations of the best procedure.

9.2 Recommendations

1. Evaluate the available hindcast wave data on the basis of spatial resolution, the length of coverage, and the accuracy of the hindcast wave heights. This evaluation must be undertaken for all populations of storms.

2. If reliable data is not available, a hindcast must be undertaken to defined specifications of the spatial resolution, the length of coverage and the accuracy of the hindcast wave heights for all populations of storms.

3. The resulting hindcast data must be separated into tropical and extratropical categories.

Evaluation of Data from Tropical Cyclones

4. The hindcast data resulting from tropical cyclones must be assessed and its importance in the determination of extreme wave statistics defined.

5. If the role of tropical cyclones is important, the spatial resolution and coverage of the hindcast data must be assessed and

procedures, compatible with the resolution and coverage, must be developed to estimate the extreme wave statistics from this group.

Evaluation of Data from Extratropical Cyclones

A systematic evaluation of the uncertainty of the procedures to estimate extreme wave statistics from the extratropical data must be undertaken.

6. In order that this evaluation can be undertaken, data processing capability must be developed:

a) to fit any selected asymptotic distribution to the hindcast data.

b) to use any selected plotting positions to plot the hindcast data.

c) to use several methods of estimating the parameters of the fitted asymptotic probability distribution.

d) to calculate control curves to define the uncertainty associated with the sample estimate of the total population.

e) to prepare plots of the fitted distribution and the observed hindcast data.

7. The above data processing capability must be used to evaluate the effects of the following physical limitations on the estimates of extreme wave statistics:

a) the definition of the threshold condition for a storm.

b) the spatial resolution of the hindcast data.

c) mixed populations of storm types.

d) the occurrence of cyclical climatic variations.

8. The assumption of stationary, independent and identical distributed variables is fundamental to the use of an asymptotic probability distribution. The violation, and consequence of such violation, of this assumption must be assessed by further study. Specifically:

a) The effects of the assumption of stationarity must be assessed after further study of probability distributions for non-stationary variables introduced in Chapter 5.2. Non-stationarity resulting from climatic changes and the growth of waves during a storm should be assessed.

b) The assumption of independence must be checked by

ensuring that all storm significant wave heights are the result of independent storms.

c) The assumption of identical distribution must be assessed after further study of the origin and storm tracks of all storms.

9. Additional studies should be undertaken to check the validity of the Weibull distribution for estimating extreme wave statistics. These studies should investigate the more general Type III distribution of the largest value.

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APPENDIX 1

THE RETURN PERIOD AND NON-ENCOUNTER PROBABILITY FOR A RANDOM NUMBER OF STORMS

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In Chapter 3 the return period and non-encounter probability were defined for two cases:

i) the structure was to be designed to withstand an event which only occurs once each year, or

ii) the structure was to be designed for storms which occur a constant number of times each year.

In reality, a random number of storms having a random intensity may occur during any given interval of time. It has been shown that the distribution of storms can be defined by a discrete probability distribution such as a Poisson or Binomial type distribution (Borgman and Resio, 1981; Teh-fu and Fang-shi, 1980; Petrauskas and Aagaard, 1971).

For a series of N random and independent storms which occur during a time interval L, each with a random intensity X associated with it, the probability that N storms occur and the largest intensity (wave height) is less than or equal to x is defined as G(Fx(x)), where the value of G(Fx(x)) is given by either the Poisson or the Binomial distribution for Fx(x). The value of Fx(x) is given by an asymptotic probability distribution of the X1...Xi...Xn observations.

The return period R (defined in this case as the average number of storms between exceedances of x) and the non-encounter probability NE(x) are related as follows:

a) The return period R is defined as:

$$R = \frac{1}{[\lambda(1 - Fx(x))]} \qquad (A1)$$

where λ = the average number of storms occuring per unit time.

b) The non-encounter probability is defined as:

$$NE(x) = \exp[-L / R] \qquad (A2)$$

where L = the life of the operation.

Equating equations A1 and A2 allows one to determine the wave height associated with a specific return period, given a defined level of risk of the event occurring.

This risk can be quantified by:

c)

$$Fx(x) = 1 + \frac{\ln [NE(x)]}{\lambda L} \qquad (A3)$$

where the value Fx(x) is given by an asymptotic probability distribution.

APPENDIX 2

THE WEIBULL DISTRIBUTION AND THE TYPE III DISTRIBUTION

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THE WEIBULL DISTRIBUTION AND THE TYPE III DISTRIBTUION

In Table 1 two forms of the Type III asymptotic distribution are introduced: the Weibull distribution and the general Type III distribution. There is a distinction between the two distributions which is important.

W. Weibull (1951) developed the Weibull distribution while investigating the fatigue strength of materials. In his investigations he was interested in the greatest stress that a sample of material could withstand without failing in fatigue. He developed the distribution from empirical reasoning to describe the probability of the smallest value of stress at which a sample would break; in other words, he was interested in the smallest value of the breaking strengths of a material. Gumbel (1958) showed from a theoretical basis that Weibull's distribution was a Type III asymptotic distribution of the smallest value in an initial limited distribution. In other words, the Weibull distibution is the asymptotic distribution of the smallest value in samples drawn from the same population. From a principle of symmetry, Gumbel also showed that a Type III asymptotic distribution of the largest values drawn from the same population existed and was of the form listed in Table 1 as the general Type III distribution. The distinciton between the two distributions is that in the case of the distribution of the smallest value, the distribution is limited on the left hand side while the distribution of the largest value is limited on the right hand side. The limit in the distribution of the smallest value therefore represents an upper bound on the stress which a material may withstand in fatigue without failing and a lower bound on the breaking strengths of the material. The Weibull distribution therefore describes the asymptotic behaviour of the smallest values of the breaking strength of a material as they approach this bound. This asymptotic behaviour towards the smallest value is of little or no interest when waves are considered unless the lower limit is assumed equal to zero and one is interested in low wave heights. The upper bound of the distribution of the largest value has a physical meaning for waves. The majority of previous studies have used the Weibull distribution of the smallest value and not the general Type III distribution of the largest value as seems more appropriate. An exception is Suarez Bores (1974) who used a general Type III distribution for the largest value. Why most studies have used the Weibull distribution is unclear.

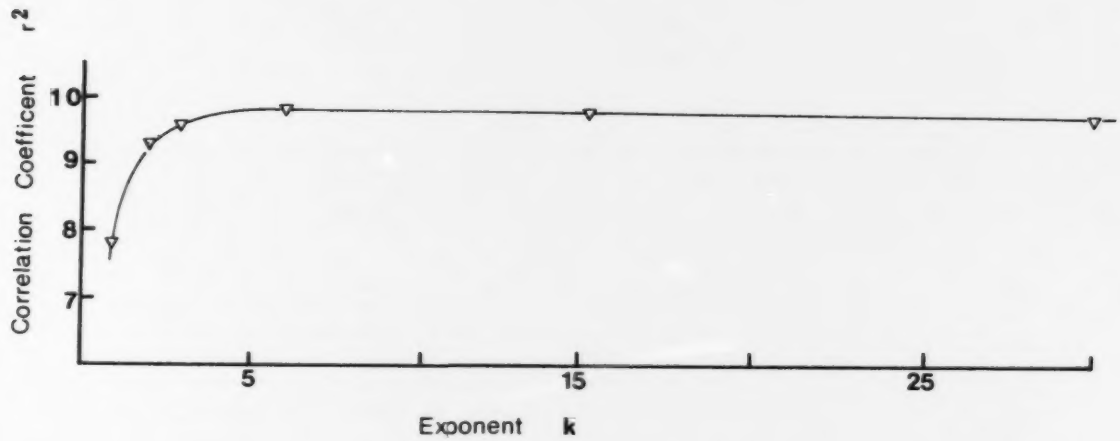
A trial application of the Type III distribution of the largest value has been undertaken for this study. The data of Petrauskas and Aagaard (1971) was used to estimate the parameters of the distribution by the method of linear regression, and the following observations were made:

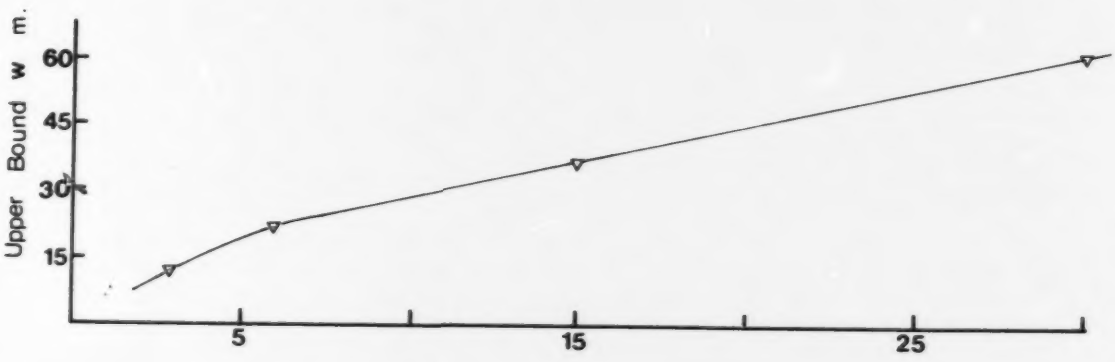
i) As the estimate of the value of the exponent k, which can be selected arbitrarily, was increased, the correlation coefficient of the fit of the distribution to the data asymptotically approached a value of 1, indicating a better fit to the data.

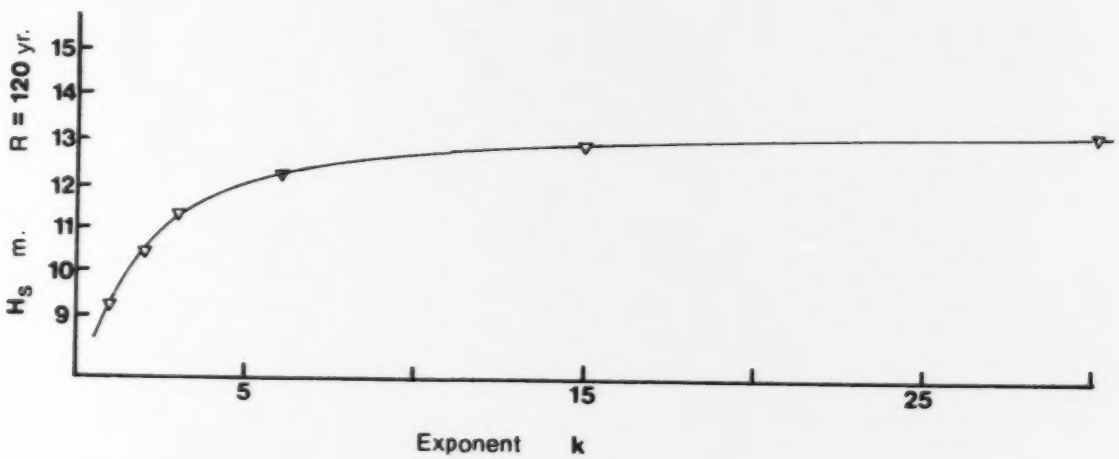
ii) as the value of the exponent increased, the magnitude of the upper bound increased and the value of the estimated extreme wave for a given return period asymptotically approached a constant value. The behaviour of the correlation coefficient, the upper bound, and the 120 year return period significant wave height are illustrated in Figure 2A.

iii) The physical meaning of the upper bound could not be evaluated because the depth of water for which the data of Petrauskas and Aagaard (1971) was derived was not available. However, for a value of the exponent at which both the correlation coefficient and the estimate of the extreme wave became asymptotic to a constant value, the magnitude of upper bound was approximately equal to the maximum depth limited wave for a depth of 80 m.

On the basis of this limited investigation, further study of the Type III distribution of the largest value should be undertaken. In particular, the behaviour of the distribution for either recorded or hindcast data for which all information is available should be undertaken. A statistical justification for the use of either the Weibull distribution (for the smallest value) or the more general Type III distribution of the largest value should be developed.







TYPE III DISTRIBUTION

FIGURE 2A

APPENDIX 3

THE CHANGE IN RETURN PERIOD WHEN THE FREQUENCY OF A GIVEN STORM TYPE INCREASES

APPENDIX 3

THE CHANGE IN RETURN PERIOD WHEN THE FREQUENCY OF A GIVEN STORM TYPE INCREASES

If only one storm type generates extreme waves, and if only one large scale atmospheric circulation pattern related to this storm type changes its probability, then the return period after the change in probability becomes:

$$T(H)_{t' \geq t} = \frac{1}{1 - [1 - [1 - F(H)]_{t < t'} B^*]}$$

where B* is the ratio of the new probability of a single storm type to the old probability of the storm type.

Example

The frequency of a single storm type which produced an extreme wave height with an initial return period of 50 years increases by 20 percent. The new return period can be calculated as:

$$T(H)_{t' \geq t} = \frac{1}{1 - [1 - [1 - .98] 1.2]}$$

where B* = 1.2

$$F(H)_{t < t'} = 0.98 = 1 - \frac{1}{50}$$

APPENDIX 4

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